Accurate Clocks and Their Applications

November 2011

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Consultant.
Much of this Tutorial was prepared while the author was employed by the
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Quartz Crystal Resonators and Oscillators
For Frequency Control and Timing Applications - A Tutorial
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Applications


The Global Positioning System (GPS) is the most precise worldwide navigation system available. It is also capable of providing nanosecond-level timing accuracies, so, it is also one of the most accurate time sources.

GPS is a satellite-based radio navigation and positioning system that is designed to provide global, all-weather, 24-hour, accurate navigation to an unlimited number of users. Each of the satellites contain atomic clocks. The satellites transmit a navigation message that provides satellite position, time, and atmospheric propagation correction data. The GPS receiver, which contains a quartz crystal clock, measures the transit time of the satellite signal and multiplies that time by the speed of light to compute range to the satellite. The satellite clocks are more accurate than the receiver clocks. Therefore, although three satellites can provide latitude, longitude and altitude, the signal from a fourth satellite is used to correct for the navigational error caused by the receiver clock's inaccuracy, i.e., the receivers calculate their \( x \), \( y \), \( z \), and \( t \) from receiving each of four satellites' \( x \), \( y \), \( z \), and \( t \). Velocity is determined from the Doppler shifts of the the transmitted carrier frequencies.


Much information is available on the Internet, e.g., see "Navstar GPS Internet Connections" at http://gauss.gge.unb.ca/GPSINTERNET.SERVICES.HTML, and "Global Positioning System Overview" by Peter H. Dana (from which the above illustration was "borrowed," with permission from Peter H. Dana, The University of Texas at Austin) at http://www.utexas.edu/depts/grg/gcraft/notes/gps/gps.html
**Electronics Applications of Clocks**

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<th>Industrial</th>
<th>Consumer</th>
<th>Automotive</th>
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</thead>
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<tr>
<td>Communications</td>
<td>Communications</td>
<td>Watches &amp; clocks</td>
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</tr>
<tr>
<td>Navigation</td>
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<td>Mobile/cellular/portable radio, telephone &amp; pager</td>
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<td>Radar</td>
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<td>Guidance systems</td>
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<td>Electronic warfare</td>
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<tr>
<td>Research &amp; Metrology</td>
<td>Digital systems</td>
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<td></td>
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<tr>
<td>Atomic clocks</td>
<td>CRT displays</td>
<td>Other medical devices</td>
<td></td>
</tr>
<tr>
<td>Instruments</td>
<td>Disk drives</td>
<td>Other digital devices</td>
<td></td>
</tr>
<tr>
<td>Astronomy &amp; geodesy</td>
<td>Modems</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Space tracking</td>
<td>Tagging/identification</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Celestial navigation</td>
<td>Utilities</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Sensors</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Example

Let R1 to R2 = 1 km, R1 to J = 5 km, and J to R2 = 5 km. Then, since propagation delay = 3.3 μs/km, \( t_1 = t_2 = 16.5 \mu s \), \( t_p = 3.3 \mu s \), and \( t_m < 30 \mu s \). Allowed clock error \( \approx 0.2 t_m \approx 6 \mu s \).

For a 4 hour resynch interval, clock accuracy requirement is: \( 4 \times 10^{-10} \)

With the availability of fast spectrum analyzers and synthesizers, it is possible to jam frequency hopping systems. If a jammer is fast enough, it can detect the frequency of transmission and tune the jammer to that frequency well before the radio hops to the next frequency. However, with a good enough clock, it is possible to defeat such “follower” jamming. As illustrated above, even a “perfect” follower jammer can be defeated if a good enough clock is available. (A perfect jammer is defined here as one that can identify the frequency of a received signal, tune a synthesizer to that frequency, and transmit the jamming signal in zero time.)

Because radio waves travel at the speed of light, the radio-to-jammer-to-radio (R1 to J to R2) and radio-to-radio (R1 to R2) propagation delays are 3.3 μs per km. Therefore, if the hopping rate is fast enough for the propagation delay difference to be greater than 1/hop-rate, i.e., if the radios can hop to the next frequency before the jamming signal reaches the receiver, then the radios are jamming-proof (for follower jammers). In the example above, the propagation delays \( t_1, t_p, \) and \( t_m \) imply that the message duration \( t_m \) be less than 30 μs. Since the clock accuracies required by frequency hopping systems are usually 10% to 20% of \( t_m \), the allowed clock error is about 6 μs. In a military environment, such accuracies can be maintained for periods of hours and longer only with atomic clocks.


In a modern battle, when the sky is filled with friendly and enemy aircraft, and a variety of advanced weapons are ready to fire from both ground and airborne platforms, positive identification of friend and foe is critically important. For example fratricide due to identification errors has been a major problem in all 20th century wars.

Current IFF systems use an interrogation/response method which employs cryptographically encoded spread spectrum signals. The interrogation signal received by a friend is supposed to result in the "correct" code being automatically sent back via a transponder on the friendly platform. The "correct" code must change frequently to prevent a foe from recording and transmitting that code ("repeat jamming"), thereby appearing as a friend. The code is changed at the end of what is called the code validity interval (CVI).

The better the clock accuracy, the shorter can be the CVI, the more resistant the system can be to repeat jamming, and the longer can be the autonomy period for users who cannot resynchronize their clocks during a mission.

Bistatic Radar

Conventional (i.e., "monostatic") radar, in which the illuminator and receiver are on the same platform, is vulnerable to a variety of countermeasures. Bistatic radar, in which the illuminator and receiver are widely separated, can greatly reduce the vulnerability to countermeasures such as jamming and antiradiation weapons, and can increase slow moving target detection and identification capability via "clutter tuning" (receiver maneuvers so that its motion compensates for the motion of the illuminator; creates zero Doppler shift for the area being searched). The transmitter can remain far from the battle area, in a "sanctuary." The receiver can remain "quiet." The timing and phase coherence problems can be orders of magnitude more severe in bistatic than in monostatic radar, especially when the platforms are moving. The reference oscillators must remain synchronized and syntonized during a mission so that the receiver knows when the transmitter emits each pulse, and the phase variations will be small enough to allow a satisfactory image to be formed. Low noise crystal oscillators are required for short term stability, atomic frequency standards are often required for long term stability.

Similar requirements exist in electronic warfare applications. The ability to locate radio and radar emitters is important in modern warfare. One method of locating emitters is to measure the time difference of arrival of the same signal at widely separated locations. Emitter location by means of this method depends on the availability of highly accurate clocks, and on highly accurate methods of synchronizing clocks that are widely separated. Since electromagnetic waves travel at the speed of light, 30 cm per nanosecond, the clocks of emitter locating systems must be kept synchronized to within nanoseconds in order to locate emitters with high accuracy. (Multipath and the geometrical arrangement of emitter locators usually results in a dilution of precision.) Without resynchronization, even the best available militarized atomic clocks can maintain such accuracies for periods of only a few hours. With the availability of GPS and using the "GPS common view" method of time transfer, widely separated clocks can be synchronized to better than 10 ns (assuming that GPS is not jammed). An even more accurate method of synchronization is "two-way time transfer via communication satellites," which, by means of very small aperture terminals (VSATs) and pseudonoise modems, can attain subnanosecond time transfer accuracies.

Another important application for low-noise frequency sources is the ELINT (Electronic INTElligence) receiver. These receivers are used to search a broad range of frequencies for signals that may be emitted by a potential adversary. The frequency source must be as noise-free as possible so as not to obscure weak incoming signals. The frequency source must also be extremely stable and accurate in order to allow accurate measurement of the incoming signal's characteristics.


Relativistic Time Effects

- Transporting "perfect" clocks slowly around the surface of the earth along the equator yields $\Delta t = -207$ ns eastward and $\Delta t = +207$ ns westward (portable clock is late eastward). The effect is due to the earth's rotation.

- At latitude $40^\circ$, for example, the rate of a clock will change by $1.091 \times 10^{-13}$ per kilometer above sea level. Moving a clock from sea level to 1km elevation makes it gain 9.4 nsec/day at that latitude.

- In 1971, atomic clocks flown eastward then westward around the world in airlines demonstrated relativistic time effects: eastward $\Delta t = -69$ ns, westward $\Delta t = +273$ ns; both values agreed with prediction to within the experimental uncertainties.

- **Spacecraft Examples:**
  - For a space shuttle in a 325 km orbit, $\Delta t = t_{\text{space}} - t_{\text{ground}} = -25 \, \mu\text{sec/day}$
  - For GPS satellites (12 hr period circular orbits), $\Delta t = +38.5 \, \mu\text{sec/day}$

- In precise time and frequency comparisons, relativistic effects must be included in the comparison procedures.


Units of Measurement Having Special Names in the International System of Units (SI)

SI Base Units

<table>
<thead>
<tr>
<th>Unit</th>
<th>Symbol</th>
<th>Base Unit</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass</td>
<td>kg</td>
<td>kilogram</td>
<td>1 kg = 1 kg</td>
</tr>
<tr>
<td>Length</td>
<td>m</td>
<td>meter</td>
<td>1 m = 1 m</td>
</tr>
<tr>
<td>Time</td>
<td>s</td>
<td>second</td>
<td>1 s = 1 s</td>
</tr>
<tr>
<td>Electric Current</td>
<td>A</td>
<td>ampere</td>
<td>1 A = 1 A</td>
</tr>
<tr>
<td>Temperature</td>
<td>K</td>
<td>kelvin</td>
<td>1 K = 1 K</td>
</tr>
<tr>
<td>Luminous Intensity</td>
<td>cd</td>
<td>candela</td>
<td>1 cd = 1 cd</td>
</tr>
<tr>
<td>Amount of Substance</td>
<td>mol</td>
<td>mole</td>
<td>1 mol = 1 mol</td>
</tr>
</tbody>
</table>

Non-SI units recognized for use with SI:
- Day: 1 d = 86400 s
- Hour: 1 h = 3600 s
- Minute: 1 min = 60 s
- Micron: 1 µm = 10^-6 m
- Millimeter: 1 mm = 10^-3 m
- Micrometer: 1 µm = 10^-6 m
- Nanometer: 1 nm = 10^-9 m
- Degree: 1 ° = (π/180) rad
- Minute of Arc: 1′ = (π/10800) rad
- Second of Arc: 1″ = (π/648000) rad
- Electronvolt: 1 eV ≈ 1.602177 × 10^-19 J
- Unified Atomic Mass Unit (u): 1 u ≈ 1.660540 × 10^-27 kg

Time interval (frequency) is the quantity that can be determined with the highest accuracy. It can be measured with an accuracy greater than 1 part in 10^13. With the help of satellites, it is possible to compare the time scales kept by the national laboratories, worldwide, to an accuracy of ~1 ns. Time, therefore, plays a central role in metrology and in the definitions of SI units.

The SI consists of seven base units and a number of derived units, as shown above. Shown on the next page are the units that do NOT depend on the unit of time.


The chart above, and the one on the next page, were provided by R.J. Douglas, National Research Council Canada, 1997.


Units of Measurement Having Special Names in the SI Units, NOT Needing Standard Uncertainty in SI Average Frequency

SI Base Units
- Mass: kilogram (kg)
- Temperature: kelvin (K)
- Amount of Substance: mole (mol)

SI Derived Units

Non-SI units recognized for use with SI:
- ton: 1 t = 10^3 kg
- degree: 1° = (π/180) rad
- minute: 1' = (π/10800) rad
- second: 1" = (π/648000) rad
- unified atomic mass unit: 1 u = 1.660540 x 10^{-27} kg
Clocks


A clock may or may not have a display. In many consumer applications, clocks display the time of day. In many other applications, clocks are used internally only; their output is typically a one-pulse-per-second (1 pps) or a time code signal which are used for sequencing or time-tagging events (see “One Pulse-Per-Second Timing Signal” and “BCD Time Code” later in this chapter.)
### Progress in Timekeeping

<table>
<thead>
<tr>
<th>Time Period</th>
<th>Clock/Milestone</th>
<th>Accuracy Per Day</th>
</tr>
</thead>
<tbody>
<tr>
<td>4th millennium B.C.</td>
<td>Sumarians divided day &amp; night into 12 equal hours</td>
<td></td>
</tr>
<tr>
<td>Up to 1280 A.D.</td>
<td>Sundials, water clocks (clipsydrae)</td>
<td></td>
</tr>
<tr>
<td>~1280 A.D.</td>
<td>Mechanical clock invented - <strong>assembly time for prayer was first regular use</strong></td>
<td></td>
</tr>
<tr>
<td>14th century</td>
<td>Invention of the escapement; clockmaking becomes a major industry</td>
<td>~15 to 30 min</td>
</tr>
<tr>
<td>~1345</td>
<td>Hour divided into minutes and seconds</td>
<td>~2 min</td>
</tr>
<tr>
<td>15th century</td>
<td>Clock time used to regulate people’s lives (work hours)</td>
<td>~1 min</td>
</tr>
<tr>
<td>16th century</td>
<td>Time’s impact on science becomes significant</td>
<td>~100 s</td>
</tr>
<tr>
<td>1656</td>
<td>(Galileo times physical events, e.g., free-fall)</td>
<td>1 to 10 s</td>
</tr>
<tr>
<td>18th century</td>
<td>First pendulum clock (Huygens)</td>
<td>10^-2 to 10^-1 s</td>
</tr>
<tr>
<td>19th century</td>
<td>Temperature compensated pendulum clocks</td>
<td></td>
</tr>
<tr>
<td>~1910 to 1920</td>
<td>Electrically driven free-pendulum clocks</td>
<td></td>
</tr>
<tr>
<td>1920 to 1934</td>
<td>Wrist watches become widely available</td>
<td></td>
</tr>
<tr>
<td>1921 to present</td>
<td>Electrically driven tuning forks</td>
<td></td>
</tr>
<tr>
<td>1949 to present</td>
<td>Quartz crystal clocks (and watches. Since ~1971)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Atomic clocks</td>
<td></td>
</tr>
</tbody>
</table>

Human beings’ use of clocks is a relatively recent phenomenon in terms of human history. Modern humans (*Homo sapiens*) are believed to have originated somewhere around 200,000 years ago. [http://en.wikipedia.org/wiki/History_of_the_Earth#2_Ma:_Human_evolution](http://en.wikipedia.org/wiki/History_of_the_Earth#2_Ma:_Human_evolution)

Sumarians divided night & day into 12 equal hours each, whose length varied as the daylight hours did. Except for astronomical purposes, equal hours were useless because people lived by the sun until the invention of mechanical clocks.

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[<http://www.horology.com/>](http://www.horology.com/)

“QUARTZ CRYSTAL RESONATORS AND OSCILLATORS
For Frequency Control and Timing Applications - A TUTORIAL”
Rev. 8.5.3.9, by John R. Vig, November 2008.
### Frequency Control Device Market
(estimates, as of ~2006)

<table>
<thead>
<tr>
<th>Technology</th>
<th>Units per year</th>
<th>Unit price, typical</th>
<th>Worldwide market, $/year</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quartz Crystal Resonators &amp; Oscillators</td>
<td>$3 \times 10^5$</td>
<td>~$1 (0.1 to 3,000)</td>
<td>~$4B</td>
</tr>
</tbody>
</table>

**Atomic Frequency Standards**
(see chapter 6)

<table>
<thead>
<tr>
<th>Technology</th>
<th>Units per year</th>
<th>Unit price, typical</th>
<th>Worldwide market, $/year</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hydrogen maser</td>
<td>~ 20</td>
<td>$100,000</td>
<td>$2M</td>
</tr>
<tr>
<td>Cesium beam frequency standard</td>
<td>~ 500</td>
<td>$50,000</td>
<td>$25M</td>
</tr>
<tr>
<td>Rubidium cell frequency standard</td>
<td>~ 50,000</td>
<td>$2,000</td>
<td>$100M</td>
</tr>
</tbody>
</table>

The estimates are based on occasional informal surveys of industry leaders. The numbers are probably accurate to a factor of two.
In an ideal resonator, the amplitude of vibration falls off approximately exponentially outside the electrodes. In a properly designed resonator, a negligible amount of energy is lost to the mounting and bonding structure, i.e., the edges must be inactive in order for the resonator to be able to possess a high Q. The displacement of a point on the resonator surface is proportional to the drive current. At the typical drive currents used in (e.g., 10 MHz) thickness shear resonators, the peak displacement is a few atomic spacings.

The peak acceleration of a point on the surface is often more than a million ‘g’s. To show this, if the displacement $u = u_0 \sin \omega t$, then, the acceleration $= \frac{d^2u}{dt^2} = -\omega^2 u_0 \sin \omega t$, and the peak acceleration $= -\omega^2 u_0$. If we assume that $u_0$ = two lattice spacings $\sim 1 \times 10^{-9}$ m, then, at 10 MHz, $\omega^2 u_0 = (2\pi \times 10^7)^2 (10^{-9}) \sim 10^6$ g.


Shown above are the bulk acoustic wave (BAW) modes of motion. For example, AT-cut and SC-cut resonators vibrate in the thickness shear mode. Above 100 MHz, overtone units that operate at a selected harmonic mode of vibration are often used (e.g., third overtone or 5th overtone). Higher than 100 MHz fundamental mode units can be manufactured by, e.g., chemical polishing (diffusion controlled wet etching), plasma etching, and ion milling techniques. Below 1 MHz, tuning forks, X-Y and NT bars (flexure mode), +6° X-cuts (extensional mode), or CT-cut and DT-cut units (face shear mode) can be used. Tuning forks have become the dominant type of low-frequency units due to their small size and low cost (see “Quartz Resonators for Wristwatches” and the following pages later in this chapter).

The velocities of acoustic waves in solids are typically ~3,000 m/s (~10^5 times the velocity of light). For the shear waves in AT-cut quartz, for example, the velocity of propagation in the thickness direction is 3,320 m/s; the fundamental mode frequency ~ v/2h, where v is the acoustic wave velocity and h is the plate thickness. (The thickness of the plate is one half the wavelength.)

Animations are courtesy of Raymond L. Filler
Above is a simplified circuit diagram that shows the basic elements of a crystal oscillator (XO). The amplifier of an XO consists of at least one active device, the necessary biasing networks, and may include other elements for band limiting, impedance matching, and gain control. The feedback network consists of the crystal resonator, and may contain other elements, such as a variable capacitor for tuning.


"Fundamentals of Quartz Oscillators," Hewlett-Packard application note AN 200-2, Hewlett-Packard Company,
<http://www.tmo.hp.com/@@2ZcNpBcQ240oRhr/tmo/Notes/English/5965-7662E.html>
Oscillation

- At the frequency of oscillation, the closed loop phase shift = 2nπ.
- When initially energized, the only signal in the circuit is noise. That component of noise, the frequency of which satisfies the phase condition for oscillation, is propagated around the loop with increasing amplitude. The rate of increase depends on the excess; i.e., small-signal, loop gain and on the BW of the crystal in the network.
- The amplitude continues to increase until the amplifier gain is reduced either by nonlinearities of the active elements ("self limiting") or by some automatic level control.
- At steady state, the closed-loop gain = 1.

See “Decay Time, Linewidth, and Q” in chapter 3 for further information on oscillator startup time.

In addition to noise, switching on the DC power supply is another oscillation trigger.


A wide temperature range XO has a typical $f$ vs. $T$ stability of $\sim 10$ to $50$ ppm. A TCXO can reduce that to $\sim 1$ ppm. An OCXO can reduce that stability to $1 \times 10^{-8}$ or better (but at the cost of much higher power consumption). High-end (SC-cut) OCXOs can stay within $1 \times 10^{-10}$ over a wide temperature range.
# Hierarchy of Oscillators

<table>
<thead>
<tr>
<th>Oscillator Type*</th>
<th>Accuracy**</th>
<th>Typical Applications</th>
</tr>
</thead>
<tbody>
<tr>
<td>Crystal oscillator (XO)</td>
<td>$10^{-6}$</td>
<td>Computer timing</td>
</tr>
<tr>
<td>Temperature compensated crystal oscillator (TCXO)</td>
<td>$10^{-8}$ to $10^{-7}$</td>
<td>Frequency control in tactical radios</td>
</tr>
<tr>
<td>Microcomputer compensated crystal oscillator (MCXO)</td>
<td>$10^{-9}$ (with $10^{-10}$ per g option)</td>
<td>Spread spectrum system clock</td>
</tr>
<tr>
<td>Oven controlled crystal oscillator (OCXO)</td>
<td>$10^{-9}$</td>
<td>Navigation system clock &amp; frequency standard, MTI radar</td>
</tr>
<tr>
<td>Small atomic frequency standard (Rb, RbXO)</td>
<td>$10^{-12}$ to $10^{-11}$</td>
<td>£C³ satellite terminals, bistatic, &amp; multistatic radar</td>
</tr>
<tr>
<td>High performance atomic standard (Cs)</td>
<td>$10^{-9}$</td>
<td>Strategic £C³, EW</td>
</tr>
</tbody>
</table>

* Sizes range from $<$5cm$^3$ for clock oscillators to $>$ 30 liters for Cs standards. Costs range from $<$5 for clock oscillators to $>$ $50,000 for Cs standards.

** Including environmental effects (e.g., -40°C to +75°C) and one year of aging.

See also chapter 7 for more detailed comparisons of various oscillators.
Quartz is the only material known that possesses the following combination of properties:

- Piezoelectric ("pressure-electric"; piezein = to press, in Greek)
- Zero temperature coefficient cuts exist
- Stress compensated cut exists
- Low loss (i.e., high Q)
- Easy to process; low solubility in everything, under "normal" conditions, except the fluoride and hot alkali etchants; hard but not brittle
- Abundant in nature; easy to grow in large quantities, at low cost, and with relatively high purity and perfection. Of the man-grown single crystals, quartz, at ~3,000 tons per year, is second only to silicon in quantity grown (3 to 4 times as much Si is grown annually, as of 1997).
Prior to ~1956, the material used for quartz resonators was natural quartz, i.e., mined quartz. Today, it is “cultured quartz,” i.e., quartz grown in factories. Although this quartz is often referred to as “synthetic quartz,” nobody has yet found a way to synthesize single crystal quartz directly from silicon and oxygen. Large quartz bars (typically ~15 cm long) of uniform size and shape are grown from small, irregularly shaped pieces of quartz (called “lascas”) by the culturing process described above. So, strictly speaking, the quartz is “cultured quartz”.

Quartz is a common material in the earth’s crust (e.g., sand is mostly quartz), however, the high purity crystals needed for quartz growing are not so common. Most of the nutrient materials used by quartz growers are mined in Brazil and the USA (near Jessieville, Arkansas).

The autoclave is a long, thick-walled ~25 to 100 cm inner diameter steel tube that can withstand the high temperatures and pressures of the growth process.

The anisotropy of quartz is discussed on the next page, and in chapter 3, where it is pointed out that the highest etching rate direction is the Z-direction. Similarly, during quartz growing, the Z-direction is the fastest direction of growth.

Polished quartz spheres, when deeply etched in concentrated HF, dissolve in a highly anisotropic manner. The partially dissolved spheres become “triangular, lenticular,” as shown above - the shape is triangular when observed along the Z-axis, and lenticular when observed along the Y-axis. The etching rate along the fastest etching direction, the Z-direction, is nearly 1000 times faster than the rate along the slowest direction, the -X direction.


The locus of zero-temperature-coefficient cuts in quartz is shown above. The cuts usually have two-letter names, where the “A” in the name indicates a temperature-compensated cut; for instance, the AT-cut is the first temperature-compensated cut discovered. The FC, IT, BT, and SBTC-cuts are other cuts along the zero-temperature coefficient locus. These cuts were studied in the past (before the discovery of the SC-cut) for some special properties, but are rarely used today. The highest-stability crystal oscillators employ SC-cut crystal units. The X, Y, and Z directions have been chosen to make the description of properties as simple as possible. The Z-axis is an axis of threefold symmetry in quartz; in other words, the physical properties repeat every 120° as the crystal is rotated about the Z-axis.


When the load capacitor is connected in series with the crystal, the frequency of operation of the oscillator is increased by a $\Delta f'$, where $\Delta f'$ is given by the equation on the previous page. When an inductor is connected in series with the crystal, the frequency of operation is decreased. The ability to change the frequency of operation by adding or changing a reactance allows for compensation of the frequency versus temperature variations of crystal units in TCXOs, and for tuning the output frequency of voltage controlled crystal oscillators (VCXO). In both, the frequency can be changed, e.g., by changing the voltage on a varactor.

Other means of temperature compensation include the use of a temperature sensitive reactance element such that the variations of the reactance with temperature compensate for the $f$ vs. $T$ variations of the resonator, and the use digital compensation techniques. The microcomputer compensated crystal oscillator (MCXO), which uses a high-accuracy digital compensation technique, is discussed in chapter 2.


The resonator, including its hermetically sealed enclosure, is made of single crystal silicon. The resonator is ~200µm on a side by ~10µm thick. The trench gap is nominally 0.4µm. Resonant frequency is ~5 MHz. Q is ~75k at room temperature. The resonator is integrated into standard silicon CMOS chips. The oscillator is, therefore, inexpensive to produce.

All oscillator frequencies are derived from the same 5MHz resonator. The oscillator is compensated by measuring the temperature with a bandgap thermometer on the CMOS die and adjusting a delta-sigma fractional PLL. The resolution is ~0.05°C, giving a frequency resolution of ~1.5ppm. The spec is +/- 100ppm and +/-50ppm depending on the grade. Carefully calibrated over temperature, a f vs. T of about +/- 5ppm can be obtained (with a cubic residue). With a simpler one-temperature calibration, typically +/-30 ppm f vs. T is obtained. Because each part is calibrated after packaging, the initial frequency offset is small, for example under about 5ppm.

The resonator is driven with five lines. The electrodes are doped silicon. At the center there is a bias contact to the resonator element at the anchor that is used to bias it, presently at 5V, the eight electrostatic drive and sense electrodes around the quad are driven at DC of zero. The four drive electrodes are divided into pairs of drive minus (on the inside of the quad) and drive plus (on the outside of the quad). Two electrodes in each pair are wired in parallel. The sense electrodes are wired in a similar way (in fact the drive and sense are interchangeable). The pairs are organized on adjacent sides so that there is minimal net drive and sense for the wineglass mode. A substrate ground connection is tied to the cover and the substrate.
What is Q and Why is it Important?

\[
Q = \frac{2 \pi}{\text{Energy stored during a cycle}}
\]
\[
= \frac{\pi}{\text{Energy dissipated per cycle}}
\]

Q is proportional to the decay-time, and is inversely proportional to the linewidth of resonance (see next page).

- The higher the Q, the higher the frequency stability and accuracy **capability** of a resonator (i.e., high Q is a necessary but not a sufficient condition). If, e.g., \( Q = 10^6 \), then \( 10^{-12} \) accuracy requires ability to determine center of resonance curve to 0.01% of the linewidth, and stability (for some averaging time) of \( 10^{-12} \) requires ability to stay near peak of resonance curve to \( 10^{-6} \) of linewidth.

- Phase noise close to the carrier has an especially strong dependence on \( Q \) (\( L(f) \propto 1/Q^4 \) for quartz oscillators).

See the next page for other definitions of Q, and see chapter 5 for additional information about the Q of quartz resonators. When the signal is decaying, as shown on the next page, the energies in the definition above are averaged over the cycle. Close to the carrier, a factor of two difference in Q results in a factor of 16 difference in phase noise.


V. B. Braginsky, V. P. Mitrofanov & V. I. Panov, **Systems with Small Dissipation**, The University of Chicago Press, 1985.

A high Q is necessary (but not sufficient) for high frequency stability - see Chapter 3 for a discussion of Q. The higher the Q, the higher the frequency stability and accuracy capability of a resonator. If, e.g., Q = 10^6, then 10^{-12} accuracy requires the ability to determine the center of the resonance curve to 0.01% of the linewidth, and stability (for some averaging time) of 10^{-12} requires the ability to stay near the peak of the resonance curve to 10^{-6} of linewidth.

A high Q is not sufficient for high stability because a high Q resonator may, for example, have a poor temperature stability. Sapphire resonators, for example, can have a very high Q, but their poor temperature stability prevents their use in clocks.

The Q, or line width of an atomic transition is determined by the observation time. The atomic resonance Qs listed above are typical values. Laser cooling of atoms can significantly extend the observation time and Q (see “Laser Cooling of Atoms” later in this chapter. Laser cooling is necessary to achieve a Cs fountain).
When an atomic system changes energy from an excited state to a lower energy state, a photon is emitted. The photon frequency $\nu$ is given by Planck's law

$$\nu = \frac{E_f - E_i}{h}$$

where $E_f$ and $E_i$ are the energies of the upper and lower states, respectively, and $h$ is Planck's constant. An atomic frequency standard produces an output signal the frequency of which is determined by this intrinsic frequency rather than by the properties of a solid object and how it is fabricated (as it is in quartz oscillators).

The properties of isolated atoms at rest, and in free space, would not change with space and time. Therefore, the frequency of an ideal atomic standard would not change with time or with changes in the environment. Unfortunately, in real atomic frequency standards: 1) the atoms are moving at thermal velocities, 2) the atoms are not isolated but experience collisions and electric and magnetic fields, and 3) some of the components needed for producing and observing the atomic transitions contribute to instabilities.

Atomic frequency standards must be understood in terms of the concepts of quantum mechanics. The properties of simple atomic systems cannot assume arbitrary values. For example, the energies of the bound states of an atomic system are constrained to discrete values called energy levels. When an atomic system changes energy from an excited state to a state with lower energy, it emits a quantity of electromagnetic energy called a photon, the frequency of which is determined by the energy difference between the two states, in accordance with Planck's law, shown above.

Atomic systems can be isolated from unwanted perturbations, which result in small sensitivities to temperature, pressure, and other environmental conditions. The low level of interaction also results in extremely sharp resonance features, and reduces errors due to imperfections in the electronics. All atoms of an element are identical, and atomic properties are time invariant, which makes it possible to build very stable devices.

Atomic frequency standards are categorized in several ways; most often, they are referred to by the type of atom: hydrogen, rubidium, or cesium. Actually, these three devices are based on the same type of atomic interaction, but there are great practical differences in their implementation. Some atomic frequency standards, called oscillators, are active, in which case the output signal is derived from the radiation emitted by the atom. Others are passive; the atoms are then employed as a discriminator to measure and control the frequency of an electronic oscillator, such as a quartz oscillator. The third classification follows the method of interaction. In atomic beams, the atoms are observed "on the fly"; they pass through the interaction region and are not used again. In contrast, storage devices contain some type of cell that holds the atoms to be observed indefinitely (ideally).

Atomic resonators are inherently noisy due to the discreet nature of atomic transitions. The short term stabilities, $\sigma_{y}(\tau)$ vs. $\tau$, vary as the square-root of the measurement interval, i.e., as $\tau^{\frac{1}{2}}$, for short intervals. This is due to the statistics of counting atomic transitions; $\sigma_{y}(\tau)$ varies as the square-root of the number of transitions. Crystal oscillators are less noisy at small $\tau$. Therefore, in all commercial atomic standards, the atomic resonator frequency is generated from the crystal oscillator's frequency (by frequency multiplication or frequency synthesis), and the crystal oscillator frequency is locked to the frequency of the atomic resonator with a servo loop time constant that is selected to provide optimum performance for the intended application. Of the many atomic transitions available, the ones selected are those which are least sensitive to environmental effects and which can be conveniently locked to the VCXO.

The atomic standard behaves as the crystal oscillator for measurement times shorter than the time constant (which, for example, is typically 100 ms to 500 ms for a Rb standard, longer in Cs standards), and it behaves as an atomic oscillator for measurement times longer than the time constant.

Since all atomic frequency standards derive their output signal from quartz oscillators, the performance of the atomic standards is significantly affected by the capabilities of the crystal oscillators. In particular, the short-term frequency stability, the vibration sensitivity, the radiation pulse sensitivity, and the sensitivity to thermal transients depend on the performance of the crystal oscillator. The atomic resonator's superior long term stability and lower sensitivity to environmental changes is used to "servo out" the crystal oscillator's aging and some of the crystal oscillator's environmental sensitivities.

------------------------


Chip Scale Atomic Clock (CSAC)

120mW power consumption
16cm³ volume (1.6” x 1.39” x 0.45”)
35g weight
±5.0E-11 accuracy at shipment
σy < 5 x 10-12 at τ = 1 hour short-term stability (Allan Deviation)
<3.0E-10/month aging rate

http://www.symmetricom.com/products/frequency-references/chip-scale-atomic-clock-csac/
Laser cooling of atoms can create atoms that move very slowly (equivalent to temperatures of microkelvins). This allows long observation times. The slow speed virtually eliminates Doppler shifts, and the long observation times allow high accuracy determinations of atomic transition frequencies, per the Heisenberg uncertainty principle, i.e., $\Delta E \Delta t \approx \hbar$ and $E = h \nu$, so $\Delta \nu \approx 1/\Delta t$. Laser cooling promises frequency accuracies of parts in $10^{16}$. The explanation of laser cooling is as follows. The numbers correspond to the numbers in the illustration above:

1. Consider two rays of light that bombard an atom. One ray travels in the same direction as the atom, the other moves in the opposite direction. The frequency of the light is slightly lower than the frequency that the atom readily absorbs.

2. From the atom’s perspective, the ray moving in the same direction as the atom is shifted down in frequency; the other ray is shifted up in frequency.

3. The atom is likely to absorb the high-frequency light but not the low. It is therefore pushed in a direction opposite its motion and slows down.

4. The emission of the absorbed light pushes the atom in some random direction, but if the process is repeated many times, the emission exerts no net force.

A fountain clock, such as NIST-F1 shown above, uses a fountain-like movement of atoms to measure frequency and time interval. First, a gas of cesium atoms is introduced into the clock's vacuum chamber. Six infrared laser beams then are directed at right angles to each other at the center of the chamber. The lasers gently push the cesium atoms together into a ball. In the process of creating this ball, the lasers slow down the movement of the atoms and cool them to temperatures near absolute zero.

Two vertical lasers are used to gently toss the ball upward (the "fountain" action), and then all of the lasers are turned off. This little push is just enough to loft the ball about a meter high through a microwave-filled cavity. Under the influence of gravity, the ball then falls back down through the microwave cavity.

The round trip up and down through the microwave cavity lasts for about 1 second. During the trip, the atomic states of the atoms might or might not be altered as they interact with the microwave signal. When their trip is finished, another laser is pointed at the atoms. Those atoms whose atomic state were altered by the microwave signal emit light (a state known as fluorescence). The photons, or the tiny packets of light that they emit, are measured by a detector.

This process is repeated many times while the microwave signal in the cavity is tuned to different frequencies. Eventually, a microwave frequency is found that alters the states of most of the cesium atoms and maximizes their fluorescence. This frequency is the natural resonance frequency of the cesium atom (9,192,631,770 Hz), or the frequency used to define the second.

The combination of laser cooling and the fountain design allows NIST-F1 to observe cesium atoms for longer periods, and thus achieve its unprecedented accuracy. Traditional cesium clocks measure room-temperature atoms moving at several hundred meters per second. Since the atoms are moving so fast, the observation time is limited to a few milliseconds. NIST-F1 uses a different approach. Laser cooling drops the temperature of the atoms to a few millionths of a degree above absolute zero, and reduces their thermal velocity to a few centimeters per second. The laser cooled atoms are launched vertically and pass twice through a microwave cavity, once on the way up and once on the way down. The result is an observation time of about one second, which is limited only by the force of gravity pulling the atoms to the ground.

As you might guess, the longer observation times make it easier to tune the microwave frequency. The improved tuning of the microwave frequency leads to a better realization and control of the resonance frequency of cesium. And of course, the improved frequency control leads to what is one of the world's most accurate clocks.

Credits
NIST-F1 was developed by Steve Jefferts and Dawn Meekhof of the Time and Frequency Division of NIST's Physics Laboratory in Boulder, Colorado. It was constructed and tested in less than four years.

The above figure, text and animation were copied from the NIST Time & Frequency Division website, http://www.boulder.nist.gov/timefreq/cesium/fountain.htm, with the permission of Steve Jefferts.
The Units of Stability in Perspective

- What is one part in $10^{10}$? (As in $1 \times 10^{-10}$/day aging.)
  - $\sim$1/2 cm out of the circumference of the earth.
  - $\sim$1/4 second per human lifetime (of $\sim$80 years).

- Power received on earth from a GPS satellite, $-160$ dBW, is as “bright” as a flashlight in Los Angeles would look in New York City, $\sim$5000 km away (neglecting earth’s curvature).

- What is $-170$ dB? (As in $-170$ dBC/Hz phase noise.)
  - $-170$ dB = 1 part in $10^{17}$ $\approx$ thickness of a sheet of paper out of the total distance traveled by all the cars in the world in a day.

The human mind is limited in its ability to understand very small and very large numbers. Above is an attempt to make the small numbers used in the frequency and time field a bit more understandable.

GPS analogy is courtesy of Raymond Filler, March 2004.
The terms accuracy, stability, and precision are often used in describing an oscillator's quality. Above is an illustration of the meanings of these terms for a marksman and for a frequency source. (For the marksman, each bullet hole's distance to the center of the target is the "measurement.") **Accuracy** is the extent to which a given measurement, or the average of a set of measurements for one sample, agrees with the definition of the quantity being measured. It is the degree of "correctness" of a quantity. **Reproducibility** is the ability of a single frequency standard to produce the same frequency, without adjustment, each time it is put into operation. From the user's point of view, once a frequency standard is calibrated, reproducibility confers the same advantages as accuracy. **Stability** describes the amount something changes as a function of parameters such as time, temperature, shock, and the like. **Precision** is the extent to which a given set of measurements of one sample agrees with the mean of the set. (A related meaning of the term is used as a descriptor of the quality of an instrument, as in a "precision instrument." In that context, the meaning is usually defined as accurate and precise, although a precision instrument can also be inaccurate and precise, in which case the instrument needs to be calibrated.)

The military specification for crystal oscillators, MIL-PRF-55310D, defines "Overall Frequency Accuracy" as "6.4.33 Overall frequency accuracy. The maximum permissible frequency deviation of the oscillator frequency from the assigned nominal value due to all combinations of specified operating and nonoperating parameters within a specified period of time. In the general case, overall accuracy of an oscillator is the sum of the absolute values assigned to the following:

a. The initial frequency-temperature accuracy (see 6.4.24).

b. Frequency-tolerances due to supply voltage changes (see 6.4.17) and other environmental effects (see 6.4.12).

Total frequency change from an initial value due to frequency aging (see 6.4.11) at a specified temperature."

The International System (SI) of units for time and frequency (the second and Hz, respectively) are obtained in laboratories using very accurate frequency standards called primary standards. A primary standard operates at a frequency calculable in terms of the SI definition of the second; for example, the duration of 9,192,631,770 periods of the radiation corresponding to the transition between the two hyperfine levels of the ground state of the cesium atom "133".
Many factors influence the frequency stability of an oscillator. Changes in the environment can cause especially large instabilities. For example, orders of magnitude (tens of dBs) changes can be observed when the phase noise of an oscillator is measured in a quiet laboratory environment, and in a vibrating environment, such as a moving vehicle.
Shown above are the major types of oscillator frequency instabilities. The pages that follow show each of the changes, and some others, in more detail.
The difference between aging and short-term instability, i.e., noise, is illustrated above. One is a systematic effect that is observed over long periods of time (days to years), whereas the other is random, observed over periods that are typically measured in fractions of a second to minutes. Over periods of hours, a combination of systematic and random effects are usually observed. The frequency vs. time characteristics over such periods often appear to be random walk (at least some of which is usually environmentally caused).

"Aging" and "drift" have occasionally been used interchangeably in the frequency control literature. However, in 1990, recognizing the "need for common terminology for the unambiguous specification and description of frequency and time standard systems," the CCIR adopted a glossary of terms and definitions. According to this glossary, aging is "the systematic change in frequency with time due to internal changes in the oscillator." Added to the definition is: "Note - It is the frequency change with time when factors external to the oscillator (environment, power supply, etc.) are kept constant." Drift is defined as "the systematic change in frequency with time of an oscillator." Drift is due to a combination of factors, i.e., it due to aging plus changes in the environment and other factors external to the oscillator. Aging is what one specifies and what one measures during oscillator evaluation. Drift is what one observes in an application. For example, the drift of an oscillator in a spacecraft is due to (the algebraic sum of) aging and frequency changes due to radiation, temperature changes in the spacecraft, and power supply changes.


http://www.itu.int/itudoc/itu-r/rec/tf/
“The noise” is a function of the averaging time (also called “measurement time” or “tau”), as is illustrated above. For the same oscillator, the fluctuations in the frequency vs. time plot measured with a 0.1 second averaging time are larger than when measured with a 1 second averaging time. Also shown are the corresponding Allan deviations.

At short averaging times, the longer the averaging time, the lower the noise, up to the “flicker floor,” i.e., for certain noise processes (see the next four pages), the hills and valleys in the frequency vs. time data average out. Longer averaging does not help when the dominant noise process is flicker of frequency. At the flicker floor, the Allan deviation is independent of averaging time. At longer averaging times, the Allan deviation increases because the dominant noise process is random walk of frequency, for which the longer the averaging time, the larger the Allan deviation.
Commercially available frequency sources cover an accuracy range of several orders of magnitude - from the simple XO to the cesium-beam frequency standard. As the accuracy increases, so does the power requirement, size, and cost. Shown above is the relationship between accuracy and power requirement. (Note that it is a log-log scale.) Accuracy versus cost would be a similar relationship, ranging from about $1 for a simple XO to about $40,000 for a cesium standard (1997 prices).
Your Head is Older Than Your Feet

- Einstein: time passes faster at higher elevations; moving clock runs slower
- Super-precise, $1 \times 10^{-17}$, atomic clocks (e.g., trapped Al ion optical clocks) can measure “time dilation” effects on a “tabletop.”
  - When one of two clocks at NIST was raised 33 cm, it ran faster than the other, as predicted by Einstein
  - When, e.g., a clock is moving at 10 km/h, it runs faster than a stationary clock, as predicted by Einstein
  - Heads age faster than feet by $\sim 0.5 \, \mu$s per human lifetime


http://www.nist.gov/pml/div688/clocks_092810.cfm
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